Light fermion masses and chiral freedom

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Abstract

Within the scenario of chiral freedom we compute the quark and lepton masses of the first two generations in terms of their chiral couplings. This allows us to make a rough estimate of the size of the chiral couplings, narrowing down the uncertainty in the chiron contribution to low energy observables, like the anomalous magnetic moment of the muon. We also extract information about the chiron mass which determines the size of possible chiron effects at the LHC.

Chiral freedom has been proposed [1] [2] as a possible solution to the gauge hierarchy problem. In a theory with chiral antisymmetric tensor fields a local mass term can be forbidden by a symmetry. The Fermi-scale of the weak interactions is then generated by dimensional transmutation. The chiral couplings between the quarks and the antisymmetric tensor fields - the chirons - are asymptotically free. Similar to the running gauge coupling in QCD they generate a non-perturbative mass scale where they grow large. If such models are viable, the LHC-experiments may be able to test this alternative to the standard model with a "fundamental" Higgs scalar.

The consistency of models with chiral antisymmetric tensors depends on properties of the ground state [3]. In particular, the generation of a non-local mass term seems to be needed. This issue is difficult to settle since it concerns the properties of the chiron propagator in a momentum range where the chiral couplings are large and non-perturbative. In the meanwhile, one may ask if such models are compatible with the present precision tests of electroweak interactions. Computations of the anomalous magnetic moment of the muon, the LEP-precision tests or interactions contributing to the b-b̄-forward-backward asymmetry all show [2] a strong dependence on the unknown size of the chiral couplings to muons or b-quarks, as well as to the infrared properties of the chiron propagator, e.g. the non-local chiron mass.

In this note we attempt a rough estimate of these unknown parameters. For this purpose we use the fact that the chiral coupling to the muon, f_μ, determines the size of the muon mass m_μ - and similar for the other "light" generations. We will see how a computation of the ratio m_μ/f_μ will give an estimate of $f_\mu f_b$. This allows for a quantitative relation between the muon anomalous magnetic moment $g - 2$ and the low momentum behavior of the chiron propagator. Even though the accuracy of these computations is limited by non-perturbative physics, the overall magnitude of some of the couplings can be constrained so that first semi-quantitative estimates of the size of possible deviations form the standard model become possible.

The chiral couplings of the antisymmetric tensor fields $\beta^\pm_{\mu\nu}$ to the quarks and leptons are given by [2], [4]

$$
\begin{aligned}
-\mathcal{L}_{ch} &= \bar{u}_R \bar{F}_U \tilde{\beta}_+ q_L - \bar{q}_L \bar{F}_U^\dagger \tilde{\bar{\beta}}_+ u_R \\
&+ \bar{d}_R \bar{F}_D \bar{\beta}_- q_L - \bar{q}_L \bar{F}_D^\dagger \beta_- d_R \\
&+ \bar{e}_R \bar{F}_L \bar{\beta}_- l_L - \bar{l}_L \bar{F}_L^\dagger \beta_- e_R,
\end{aligned}
\tag{1}
$$

with

$$
\beta_\pm = \frac{1}{2} \beta^\pm_{\mu\nu} \sigma^{\mu\nu} \ , \ \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu].
\tag{2}
$$

1

With respect to the electroweak gauge symmetry the chirons $\beta^{\pm}$ transform as doublets with hypercharge $Y = 1$, similar to the Higgs doublets. (For details see [2].) The model has a discrete symmetry $\beta^- \to -\beta^-$.

The chiral couplings are organized in 3×3-matrices $F_{U,D,L}$. Similar to the Yukawa couplings in the standard model they carry the information about (approximate) flavor symmetries and mixings. Indeed, we can choose phase conventions for the fermions such that $F_{U,D,L}$ are diagonal and real. All information on flavor mixing and CP-violating phases appears then only in the interactions of the W-boson in the form of the usual CKM-matrix [5]. These effects are small and will be neglected for the purpose of this note. In this limit separate conserved flavor symmetries are present, like a conserved L_μ for the lepton number of muons and muon-neutrinos. This implies that also the mass matrices for the quarks and leptons must be real and diagonal. We will choose conventions where all mass eigenvalues are positive. With these conventions the sign of the chiral couplings matters - indeed the sign of m_μ/f_μ is, in principle, measurable and cannot be changed by chiral phase rotations. For example, this sign will determine the sign of $g - 2$.

In the limit of vanishing f_μ the model exhibits an additional axial symmetry for the muons. (Effects of axial anomalies are negligible for this note.) In turn, this implies a vanishing m_μ. We conclude $m_\mu = h f_\mu$, where the proportionality constant h depends on the various couplings of the model and needs to be computed. Since h can only involve even powers of f_μ we will restrict the computation of m_μ to the order linear in f_μ. Of course, this is only justified if f_μ turns out to be small enough. We will present our computation in the form of a loop calculation. We will argue below that this form is indeed justified even in a non-perturbative context, if appropriate effective couplings and propagators are used. The loop calculation should therefore give a reasonable estimate of the orders of magnitude involved.

In linear order in the chiral coupling f_μ the muon mass gets contributions from the diagrams shown in fig. 1. In turn, the effective mixing between β^- and the gauge bosons $\gamma, Z, W^\pm$ involves a loop with the exchange of a virtual b-quark, as shown in fig. 2. Thus the muon mass arises effectively in two loop order. Parametrically one finds

$$m_\mu = c_\mu f_\mu f_b e^2 m_b \tag{3}$$

and our task is the computation of the proportionality coefficient c_μ. One expects that c_μ depends on the chiron mass m_- and the couplings, masses and mixings for the W and Z bosons.

We start with the contribution involving a photon line in the loop. It is convenient to organize the computation in two steps. We first compute the
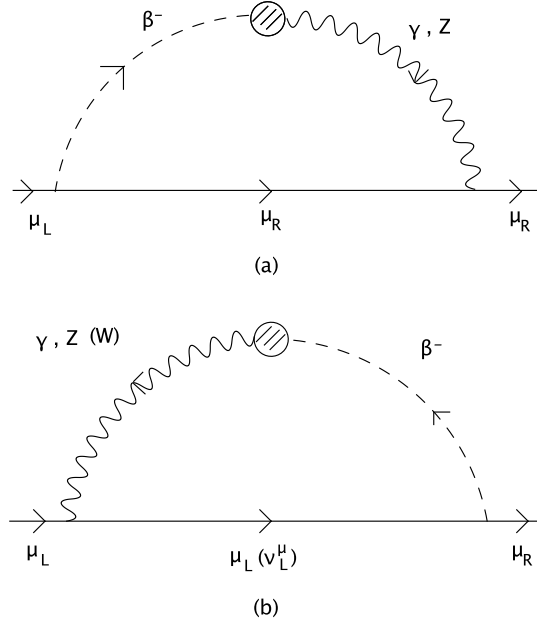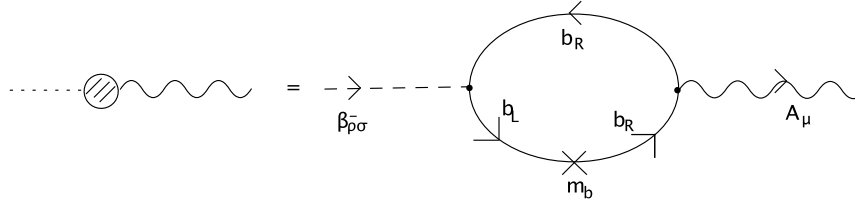
Figure 1: Loop contributions to the muon mass

Figure 2: Chiron-photon mixing

effective momentum dependent chiral vertex of the photon to the charged leptons and down quarks

$$
\begin{aligned}
\Gamma_{\gamma,\text{ch}1} \;=\;& \int_p \int_q \alpha_\gamma(p,q)\left\{\bar{e}_R(q+p)F_L\sigma^{\mu\nu}e_L(q)\right. \\
& \left. +\bar{d}_R(q+p)F_D\sigma^{\mu\nu}d_L(q)\right\}F_{\mu\nu}(p) + \text{h. c.}
\end{aligned}
\tag{4}
$$

Here $e_R = (e_R, \mu_R, \tau_R)$ is a vector with three flavor components for the right handed leptons and similar for e_L, d_R and d_L. The 3×3 matrices F_L, F_D denote the renormalized chiral couplings to the antisymmetric tensors and we may include their momentum dependence if needed. For our specific case of the muon we replace $\bar{e}_R F_L \sigma^{\mu\nu} e_L$ by $f_\mu \bar{\mu}_R \sigma^{\mu\nu} \mu_L$.

Secondly, we evaluate the diagrams in fig. 3, where the circles denote the effective chiral photon vertex (2).
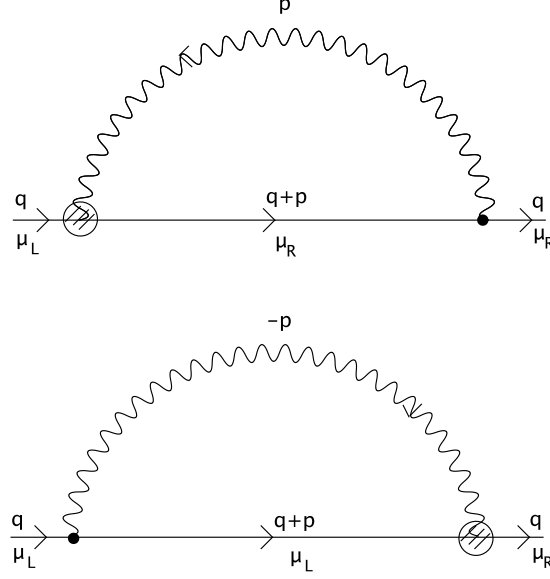
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Figure 3: Loops with effective chiral photon vertex

This yields, for $q^2 \to 0$ and with $\sigma_{\mu\nu}\sigma^{\nu\mu} = -6$, $f_\mu = f_\mu(p = 0)$,

$$
\begin{aligned}
m_\mu^{(\gamma)} &= 2e \int_p f_\mu(p)\alpha_\gamma(p,0)\frac{p_\mu p_\rho}{p^4}\left\{\sigma^{\mu\nu}\gamma^\rho\gamma_\nu - \gamma_\nu\gamma^\rho\sigma^{\mu\nu}\right\} \\
&= -6ief_\mu \int_p \frac{\alpha_\gamma(p,0)}{p^2}\frac{f_\mu(p)}{f_\mu} .
\end{aligned}
\tag{5}
$$

In lowest order in the mixing between the chiral tensors and the gauge bosons the effective chiral vertex of the photon is composed of the vertex for the coupling of the chiral tensor to the muon ($\sim f_\mu\sigma^{\mu\nu}$), the mixing between the chiral tensor and the photon ϵ_γ^-, and the propagator of the chiral tensor, as shown in fig. 4. The dominant momentum dependence of $\alpha_\gamma(p, q = 0) = \alpha_\gamma(p)$ arises from the propagator of the chiral tensor field B^{-0}

$$
\alpha_\gamma(p) = -\frac{\epsilon_\gamma^-(p)}{2\sqrt{2}}\frac{c_{\beta,\gamma}}{Z_-(p)p^2 + m_-^2(p)} .
\tag{6}
$$

For simplicity we neglect the mixing of the real part of the neutral component of β, i.e. $(\beta^{-0} + (\beta^{-0})^*)/\sqrt{2}$, with other neutral components of the chiral tensor or with the Z-boson. In this approximation one has $c_{\beta,\gamma} = 1$, whereas more generally the additional mixing effects can be incorporated in $c_{\beta,\gamma}$.
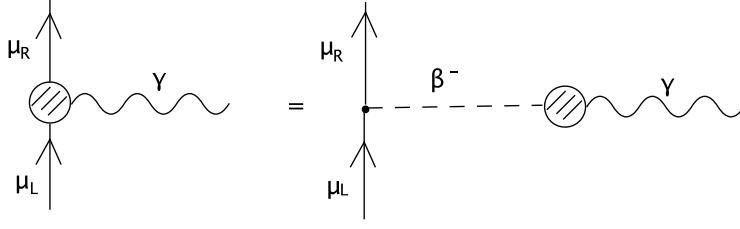
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Figure 4: Effective chiral vertex of the photon

The photon-chiron mixing coefficient $\epsilon_\gamma^-(p)$ corresponds to a piece in the effective action

$$\Gamma_m = -\frac{1}{2\sqrt{2}} \int_p \epsilon_\gamma^-(p) \left[\beta_{\mu\nu}^{-0}(-p) + \beta_{\mu\nu}^{-0}(p)^*\right] F^{\mu\nu}(p). \tag{7}$$

One finds

$$\epsilon_\gamma^-(p) = \frac{8ie}{\sqrt{2}} m_b(p) f_b(p) I_1(p) \tag{8}$$

where we approximate the momentum integral in the loop similar to [2]

$$
\begin{aligned}
I_1(p) &\approx \int_q \left[\left(q + \frac{p}{2}\right)^2 + m_b^2\right]^{-1} \left[\left(q - \frac{p}{2}\right)^2 + m_b^2\right]^{-1} \\
&\approx \frac{i}{16\pi^2} \ln\left(\frac{\Lambda_b^2 + p^2}{m_b^2 + p^2/8}\right).
\end{aligned}
\tag{9}
$$

(For this approximation to I_1 we have restricted the euclidean momentum integral in the b-quark loop (fig. 2) to $q^2 < \Lambda_b^2 + p^2$, taking into account that $m_b(q + p)/m_b(p)$ vanishes for large loop momenta q.) Combining the different pieces yields

$$\alpha_\gamma(p) = \frac{c_{\beta,\gamma}}{8\pi^2} \frac{e\, m_b(p) f_b(p)}{Z_-(p)p^2 + m_-^2(p)} \ln\frac{\Lambda_b^2 + p^2}{m_b^2 + p^2/8}. \tag{10}$$

We next evaluate the loop integral for $m_\mu^{(\gamma)}$ (5) by analytical continuation of q_0 to euclidean signature. The momentum dependence of the logarithmic factor $I_1(p)$ in $\alpha_\gamma(p)$ is weak and we will replace $I_1(p)$ by a constant $I_1(p^2 = m_-^2)$. For a computation of $m_\mu^{(\gamma)}$, cf. eq. (5), we therefore need to evaluate the integral

$$I_2 = i \int_p^{(E)} \frac{1}{p^2(Z_-(p)p^2 + m_-^2(p))} \frac{f_\mu(p)}{f_\mu} \frac{f_b(p)}{f_b} \frac{m_b(p)}{m_b} \tag{11}$$

5

with $f_{\mu,b} = f_{\mu,b}(p = 0)$, $m_b = m_b(0)$. Here $\int_p^{(E)} = \int^{(E)} \frac{d^4 p}{(2\pi)^4}$ denotes the integral with euclidean signature. The precise meaning of the wave function renormalization $Z_-(p)$ and the couplings $f_{\mu,b}(p)$ depends on the normalization of the chiral tensor field. It is convenient to adopt a different normalization for large and small p^2. For the range $p^2 < m_-^2$ we work in a momentum independent normalization for the chiral tensor field, where $Z_-(p^2 < m_-^2)$ may depend on p, with $m_-^2 = m_-^2(p^2 = m_-^2)$. For $p^2 > m_-^2$ we adopt a momentum dependent normalization, keeping the 'kinetic term' $\sim p^2$ fixed in the inverse propagator, $Z_-(p^2 \geq m_-^2) = 1$.

The momentum dependence of $f_b(p)$ and $m_b(p)$ will ensure the ultraviolet finiteness of I_2. Let us first consider the range $m_b^2 < p^2 < m_-^2$. For a momentum independent normalization of the chiral tensors the effects from the anomalous dimension of these fields should not be included in the momentum-dependence of the chiral couplings. Then the chiral couplings $f_{\mu,b}(p)$ are essentially constant for $p^2 < m_-^2$ and we may associate $f_{\mu,b} = f_{\mu,b}(p^2 = m_-^2)$. With this convention the anomalous dimension induced by b-quark loops shows only up in the kinetic term for the chiral tensors

$$Z_-(p) = \frac{f_b^2 + \frac{1}{3}f_\tau^2}{4\pi^2} \ln \frac{m_-^2}{p^2 + m_b^2} \quad \text{for} \quad p^2 < m_-^2 . \tag{12}$$

Since for $p^2 \ll m_-^2$ the inverse chiron-propagator is dominated by m_-^2 the momentum dependence of $Z_-(p)$ is actually not very important. We neglect it and set $Z_- = 1$.

On the other hand, for $p^2 > m_-^2$ the inverse chiron propagator is dominated by p^2 and the momentum dependence of $m_-^2(p)$ becomes unimportant. We may effectively replace $m_-^2(p)$ by the constant m_-^2 which corresponds to a suitable average value of $m_-^2(p)$ for $p^2 \approx m_-^2$. For $p^2 > m_-^2$ the chiral coupling of the muon is enhanced

$$\frac{f_\mu(p)}{f_\mu} = \left(\frac{p^2}{m_-^2}\right)^{\frac{\bar{f}^2(p)}{8\pi^2}} ,$$

$$\bar{f}^2(p) = \int_{\ln m_-}^{\ln |p|} dt \left(f_b^2(t) + \frac{1}{3}f_\tau^2(t) \right) / \ln \frac{|p|}{m_-} . \tag{13}$$

We observe that $f_b(p)$ decreases much faster such that the factor $f_\mu(p)f_b(p)$ decreases for increasing p^2. Furthermore, $m_b(p)$ is expected to decrease and to vanish for sufficiently large p^2. We combine these effects in an effective ultraviolet cutoff, restricting the integral I_2 to $p^2 < \tilde{\Lambda}^2$, and approximate

$$I_2 = \frac{i}{16\pi^2} \ln \frac{\tilde{\Lambda}^2 + m_-^2}{m_-^2} . \tag{14}$$

6

Combining the different pieces yields

$$m_\mu^{(\gamma)} = \frac{3c_{\beta,\gamma}}{64\pi^4} \ln\left[8\left(1 + \frac{\Lambda_b^2}{m_-^2}\right)\right] \ln\left(1 + \frac{\tilde{\Lambda}^2}{m_-^2}\right) e^2 f_b f_\mu m_b \ . \qquad (15)$$

Our rather crude discussion reveals that this result still has substantial uncertainties, cast here into the form of effective cutoffs Λ_b and $\tilde{\Lambda}$ and the mixing coefficient $c_{\beta,\gamma}$. Nevertheless, $c_{\beta,\gamma}$ is expected to be of the order of magnitude one. The dependence on Λ_b and $\tilde{\Lambda}$ is only logarithmic.

It is useful to trace the origin of the different factors appearing in the ratio m_μ/m_b. For this purpose we introduce the coefficient

$$\kappa_\mu = \frac{m_\mu}{m_\mu^{(\gamma)}} \qquad (16)$$

for the ratio between the total muon mass and the contribution from the photon loop. We write

$$
\begin{aligned}
m_\mu &= \kappa_\mu e \, f_\mu \hat{\alpha}_\gamma b_\mu \ , \\
\hat{\alpha}_\gamma &= \frac{c_{\beta,\gamma}}{8\pi^2} e f_b m_b \ln\left[8\left(1 + \frac{\Lambda_b^2}{m_-^2}\right)\right] \ , \\
b_\mu &= \frac{3}{8\pi^2} \ln\left(1 + \frac{\tilde{\Lambda}^2}{m_-^2}\right),
\end{aligned}
\qquad (17)
$$

which separates the factors arising from the chiral coupling of the photon $(\hat{\alpha}_\gamma)$ and the remaining loop integral (b_μ). Here $f_\mu \hat{\alpha}_\gamma/(m_-^2(p) + Z_- p^2)$ measures the strength of the anomalous muon-photon coupling $f_\mu(p)\alpha_\gamma(p)$, suitably averaged in the relevant momentum range. Since the coefficient b_μ is positive, the sign of $f_\mu \hat{\alpha}_\gamma/m_\mu = (eb_\mu \kappa_\mu)^{-1}$ depends only on the sign of κ_μ. This sign will determine the sign of the correction $\Delta(g-2)$ to the anomalous magnetic moment of the muon. In terms of the factors (17) the coefficient c_μ, in eq. (3), reads

$$c_\mu = \frac{\kappa_\mu b_\mu \hat{\alpha}_\gamma}{e f_b m_b} \ . \qquad (18)$$

What remains is the computation of κ_μ. For this purpose we have to compare the weight of the diagrams involving the exchange of Z-or W-bosons with the photon exchange diagram. The effective coupling of the Z-boson to the muon has the same chiral structure as for the photon. We only have to replace the photon-chiron mixing by the Z-chiron mixing. Working again

in the approximation where $(\beta^{-0} + \beta^{-0*})/\sqrt{2}$ dominates, we find a relative factor

$$f_{Z,b} = \frac{1}{\cos\vartheta_W \sin\vartheta_W}\left(\frac{3}{4} - \sin^2\vartheta_W\right) \tag{19}$$

from the coupling of the Z-boson to the b-quark in the diagram corresponding to fig. 2. (Only the vector coupling of Z contributes, not the axial vector.) A similar factor arises from the coupling to the muon (again only the vector coupling matters)

$$f_{Z,\mu} = \frac{3}{4\cos\vartheta_W \sin\vartheta_W}(1 - 4\sin^2\vartheta_W). \tag{20}$$

For the diagrams involving the exchange of the W-boson we first evaluate the effective chiral vertex of the W-boson with the leptons ($W^\mu = W^{+\mu} = (W_1^\mu - iW_2^\mu)/\sqrt{2}$, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$)

$$\Gamma_{W,ch1} = \int_p \int_q \alpha_W(p,q)\bar{e}_R(q+p)F_L\sigma^{\mu\nu}\nu_L(q)W_{\mu\nu}(p) + h.c. \tag{21}$$

A computation similar to the chiron-photon mixing yields

$$\alpha_W(p,q) = \alpha_\gamma(p,q)f_{W,b}\frac{c_{\beta,W}}{c_{\beta,\gamma}} ,$$
$$f_{W,b} = -\frac{3}{2\sqrt{2}\sin\vartheta_W}. \tag{22}$$

Here $f_{W,b}$ accounts for the different coupling of the W-boson to the b-quark as compared to the photon. The factor $c_{\beta,W}$ accounts again for mixing effects in the chiron propagator and incorporates the possibility that the mass of the charged chiron β^{-+} may differ from the neutral chiron β^{-0}. In the W-exchange diagram in fig. 1b we pick up an additional factor $f_{W,\mu}$ from the $W\mu\bar{\nu}$ vertex

$$f_{W,\mu} = -\frac{1}{2\sqrt{2}\sin\vartheta_W}. \tag{23}$$

Summing up all diagrams this yields for κ_μ

$$\kappa_\mu = 1 + f_{Z,b}f_{Z,\mu}\frac{c_{\beta,Z}}{c_{\beta,\gamma}} + f_{W,b}f_{W,\mu}\frac{c_{\beta,W}}{c_{\beta,\gamma}}. \tag{24}$$

If we assume for a moment $c_{\beta,Z} = c_{\beta,W} = c_{\beta,\gamma}$ we obtain

$$\bar{\kappa}_\mu = \frac{11 - 33\sin^2\vartheta_W + 16\sin^4\vartheta_W}{8\cos^2\vartheta_W \sin^2\vartheta_W} \approx 3.5 \tag{25}$$

8

and find a moderate enhancement as compared to the pure photon-exchange contribution. This is mainly due to the dominant W-exchange diagram. Since the W-exchange contribution can easily overweigh the photon exchange, an assessment of the sign of κ_μ requires an investigation of the possible sign of $c_{\beta,W}/c_{\beta,\gamma}$. We will turn to this question below.

An important ingredient for our computation of m_μ/m_b, namely the effective chiral coupling of the photon (4), is accessible to measurement for low momenta p^2. Indeed, we note an interesting relation between our computation of the relation between m_μ and f_μ on the one side and a contribution to the anomalous magnetic moment of the muon on the other side. The chiral coupling of the photon in eq. (4) directly contributes to $g - 2$ [2]

$$\Delta(g - 2) = -\frac{8m_\mu}{e} f_\mu \alpha_\gamma(0) . \tag{26}$$

Using the experimental bounds on $\Delta(g - 2)$ this will allow us to restrict the allowed range for certain parameters. In particular, the product $f_\mu f_b$ can be eliminated in favor of $\Delta(g - 2)$. In order to exploit the relation (26) we may write eq. (17) as

$$m_\mu = e f_\mu \alpha_\gamma(0) m_-^2(0) H \kappa_\mu \tag{27}$$

with

$$H = \frac{\hat{\alpha}_\gamma}{\alpha_\gamma(0) m_-^2(0)} b_\mu = K b_\mu. \tag{28}$$

Here we recall that $\hat{\alpha}_\gamma$ is essentially the chiral photon coupling with the chiron propagator amputated and evaluated for an appropriate momentum $p^2 \approx m_-^2$. The factor K therefore accounts for the momentum dependence of the effective anomalous photon vertex. We write it in the form

$$K = \frac{I_1(p^2 = m_-^2)}{I_1(p^2 = 0)} \tilde{f}_K = \frac{\ln 8 \left[1 + \Lambda_b^2/m_-^2\right]}{\ln[\Lambda_b^2/m_-^2]} f_K \tag{29}$$

where the second equation uses our simple estimates of the relevant momentum integrals, cf. eq. (9). The factor f_K contains the uncertainties in this estimate and the possibility of a momentum dependence of $c_{\beta,\gamma}$. We will take $f_K = 1$. In other words, the factor K reflects the fact that $\Delta(g - 2)$ measures $\alpha_\gamma(p^2)$ for $p^2 \to m_\mu^2$ where the effective chiron propagator is constant $\sim c_{\beta,\gamma}(0)/m_-^2(0)$. In contrast, the momentum integral in the loop in fig. 2 covers effectively the range $m_b^2 < q^2 < \Lambda_b^2$. Here the integral for $\alpha_\gamma(p^2 \gg m_b^2)$ is cut off by $p^2/8$. One therefore expects $K < 1$ and we will take $K \approx 0.3$.

We can now use eq. (27) in order to eliminate $f_\mu \alpha_\gamma(0)$ in eq. (26)

$$\Delta(g - 2) = -\frac{8m_\mu^2}{H \kappa_\mu e^2 m_-^2(0)} \approx -2.5 \cdot 10^{-6} \frac{\bar{H}}{H} \frac{\bar{\kappa}_\mu}{\kappa_\mu} \frac{1 TeV^2}{m_-^2(0)}, \tag{30}$$

9

with $\bar{H}\bar{\kappa}_\mu = 0.1$ reflecting our crude estimates. The experimental information on $\Delta(g-2)$ can directly be used for limiting the acceptable range of $m^2_-(0)$. Much of the uncertainties in the estimates of [2], which have involved the unknown chiral couplings f_μ and f_b, are now eliminated by our computation of m_μ/m_b where the same couplings enter. We may take the present difference between the experimentally measured [6] mean value of $g-2$ and the mean value of theoretical estimates [7] as a guide, $\Delta(g-2) = 6 \cdot 10^{-9}$. For $H\kappa_\mu = -\bar{H}\bar{\kappa}_\mu$ eq. (30) yields $m_-(0) = 20 TeV$. This value is considerably larger than the top quark mass. A large ratio $m_-(0)/m_t$ will require an explanation. Unfortunately, a large chiron mass would also render the experimental detection of effects mediated by the chiral tensors much harder.

We may also us the relation (17) for an estimate of the product of chiral couplings $f_\mu f_b$. For our crude estimate we find

$$\frac{m_\mu}{m_b} = 2 \cdot 10^{-3} f_\mu f_b \; \frac{\kappa_\mu}{\bar{\kappa}_\mu}, \;\; f_\mu f_b \approx 5 \frac{\bar{\kappa}_\mu}{\kappa_\mu}. \tag{31}$$

The rather large value of $f_\mu f_b$ is at the origin of the high value of $\bar{H}$ and therefore the large value of $m_-(0)$. For $f_b^2 \approx 8\pi$ one finds $f_\mu^2/(4\pi) \approx 0.1$, justifying the expansion linear in f_μ.

Substantial uncertainties remain in our computation. They mainly concern two issues. One concerns the estimate of the momentum dependence of the effective chiral photon coupling. In particular, the momentum dependence of the chiron propagator for small p^2 can be substantial, with $m^2_- \ll m^2_-(0)$. It has been argued in [3] that the stability of chiral tensor models requires an asymptotic vanishing of the inverse chiron propagator $m^{-2}_-(p)$ for $p^2 \to 0$, at least $\sim p^2$. In this case a substantially higher value of $m^2_-(0) \widehat{=} m^2_-(p^2 = m^2_\mu)$ as compared to $m^2_- \widehat{=} m^2_-(p^2 = m^2_-)$ is suggested. This effect could suppress the contribution to $\Delta(g-2)$ to a very small or even practically unoberservable level, even for values $m^2_- \lesssim 1 TeV^2$. Rather strong observable chiron-effects at the LHC could be compatible with very small $\Delta(g-2)$.

The other uncertainty arises from the relative weight of W- and Z-exchange contributions to m_μ/m_b as compared to the photon contribution, casted here in the coefficient κ_μ. In particular, this concerns the sign of κ_μ and therefore the sign of $\Delta(g-2)$. Any enhancement in H or κ_μ would result in a corresponding decrease of $m^2_-(0)$. For both cases the main uncertainty arises from the "mixing coefficients" $c_{\beta,\gamma}, c_{\beta,W}, c_{\beta,Z}$ which multiply the effective chiral vertices of the gauge bosons. For fixed chiral vertices the uncertainties of the evaluation of the loop diagram in fig. 3 seem only moderate, since the dependencies on effective infrared and ultraviolet cutoffs are only logarithmic.

At this point a few general remarks on the status of our loop computation in a non-perturbative setting seem appropriate. Indeed,our computation contains an essentially non-perturbative part, namely the effective chiral vertices of the gauge bosons. This is combined with the perturbative evaluation of the loop diagrams in fig. 3, resulting in eq. (3), and the factor b_μ in eq. (17). While the uncertainties in b_μ are only logarithmic, the estimate of the coefficients $c_{\beta,\gamma}, c_{\beta,Z}$ and $c_{\beta,W}$ is very sensitive to the non-perturbative physics.

The diagrams in fig. 1 are actually the only contributions to m_μ in linear order in f_μ. Indeed, the conserved quantum number L_μ (in the limit of unit CKM-matrix) implies that for arbitrary diagrams the muon line connecting the in- and out-going muon must be continuous. Since for $f_\mu = 0$ the muon mass vanishes, one vertex must involve the chiron and be proportional to $f_\mu \sigma^{\mu\nu}$. On the other hand, this vertex must be reconnected to the muon line. For a contribution to $\bar{\mu}_R \mu_L$ with chirality flip, this necessarily involves a gauge boson vertex $\sim \gamma^\mu$. Indeed, a contribution to m_μ must involve an odd number of vertices with chirality flip along the muon line. A second vertex involving a chiron would induce a further chirality flip and an additional power of f_μ. Also a possible vertex with a composite scalar field would induce a further chirality flip. It would therefore have to occur in an even number. (Also an even number of further gauge boson vertices are possible. This leads to radiative corrections to the diagram of fig. 1, that may be neglected.)

We conclude that all contributions to m_μ which are linear in f_μ must involve a mixing between chiral tensors and gauge bosons. Summing over all fields with a nonvanishing chiral vertex to the muon one arrives at the description in terms of the effective chiral vertices of the gauge bosons. The propagation and mixings of the chirons are affected strongly by non-perturbative effects, such that the effective chiral vertices of the gauge bosons and their momentum dependence are essentially non-perturbative.

An important non-perturbative effect concerns the mixing between the antisymmetric tensors $\beta_{\mu\nu}^+$ and $\beta_{\mu\nu}^-$ which is induced by the electroweak symmetry breaking [2]. For the charged chirons such a mixing appears in the form of an effective Lagrangian

$$- \mathcal{L}_{M\beta} = \frac{\tilde{m}^2}{4} \beta_-^{\mu\nu*} \beta_{+\mu\nu} + h.c. \tag{32}$$

This generates an additional contribution to the chiral vertex of the W-bosons to the muon, according to fig. 5. The mixing between β^+ and β^- is small, $\tilde{m}^2/(m_+^2 - m_-^2) \ll 1$. The off-diagonal mass term $\tilde{m}^2$ involves the vacuum expectation values of the two composite Higgs scalars that are responsible for the top and bottom masses, respectively. Typically, it is
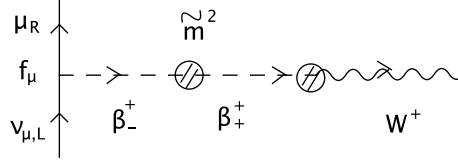
11

Figure 5: Mixing contribution to the effective chiral vertex of the W-boson.

therefore substantially smaller than the mass term m_+^2 for the chirons $\beta_+^{\mu\nu}$. On the other hand, the effective mixing between β_+^+ and W^+ is enhanced by a factor $-f_t m_t/(f_b m_b)$ as compared to the $\beta_-^+ - W^+$-mixing, as shown in the comparison of the two graphs in fig. 6.
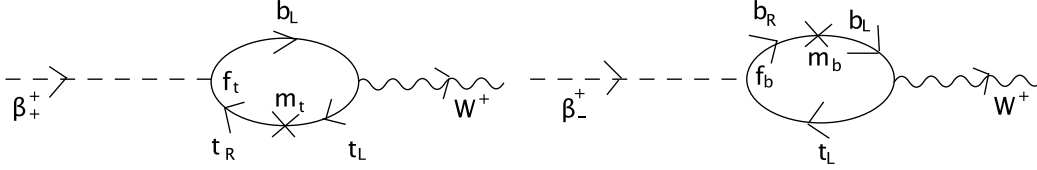

Figure 6: Chiron-W-boson mixing

This enhancement is very substantial and may overcompensate the suppression $\sim \tilde{m}^2/(m_+^2 - m_-^2)$. In this case the diagram in Fig. 5 dominates the effective chiral vertex between the W-boson and the muon. As a result, the W-contribution to κ_μ is multiplied by a factor

$$c_{\beta,W} \approx 1 + f_t m_t \tilde{m}^2 / \left[f_b m_b (m_+^2 - m_-^2) \right] \tag{33}$$

and can become large. Its sign depends on the sign of $\tilde{m}^2$ and f_t/f_b.

Similar $\beta_+^0 - \beta_-^0$-mixing contributions are present for the effective chiral vertices of the photon and the Z-boson. Now $\tilde{m}^2$ is replaced by an off diagonal mass term $\tilde{m}_0^2$ for the neutral chiral tensors, and correspondingly for m_+^2 and m_-^2. For the photon one also has an additional factor (-2) reflecting the different electric charges of the top and bottom quarks [2].

The size of $\tilde{m}^2$ and $\tilde{m}_0^2$ could be quite different. This may be seen by observing that we can write down a one loop expression for $\tilde{m}^2$, shown in fig. 7. No such graph exists for the neutral chirons. We recall, however, that the effective chiron propagator is a non-perturbative quantity such that loop-considerations give at best some qualitative insight.

Our discussion shows that a negative value of κ_μ seems well possible. This would imply $\Delta(g-2) > 0$. Unfortunately, this investigation has also revealed a substantial uncertainty in the mixing coefficients $c_{\beta,W}$ etc. The
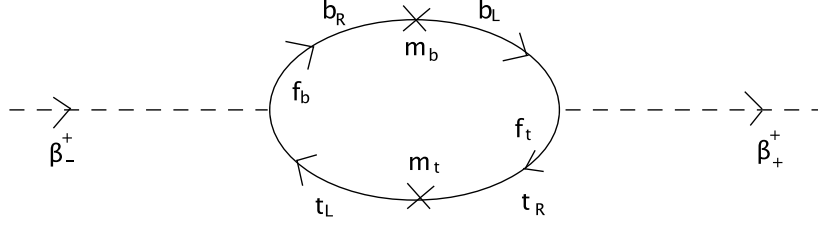
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Figure 7: Loop contribution to charged chiron mixing

corresponding uncertainty in the absolute size of κ_μ directly affects the estimates of $m_-^2(0)$ (20) and $f_\mu f_b$ (21). For $\kappa_\mu \approx -100$ we would arrive at $m_-(0) \approx 4TeV$, $f_\mu f_b \approx 0.2$. Clearly, further quantitative precision needs a better understanding of the chiron propagator.

In conclusion, we have established the mechanism how the masses of the light generations of quarks and charged leptons are generated within the scenario of chiral freedom. With minor modifications our calculations can be used to compute m_s/m_b or m_c/m_t in terms of f_s and f_c. Due to the linearity in the chiral couplings one finds for the first generation the relations

$$\frac{m_e}{m_\mu} = \frac{f_e}{f_\mu} \ , \ \frac{m_u}{m_c} = \frac{f_u}{f_c} \ , \ \frac{m_d}{m_s} = \frac{f_d}{f_s}. \tag{34}$$

The mechanism for the mass generation of the b-quark (and probably also the tau-lepton) is different. Even in presence of a nonvanishing top quark mass it requires the spontaneous breaking of the discrete symmetry.

Despite substantial non-perturbative uncertainties concerning the chiron propagator or, correspondingly, the effective chiral vertex of the gauge bosons, we have gained first estimates of the size of the chiral couplings, in particular $f_\mu f_b$. This helps to relate observable quantities, like the anomalous magnetic moment of the muon, to the overall size of the chiron propagator or the chiron mass m_-. Knowledge of the latter is crucial for the estimate of possible new LHC-physics arising from the chirons. At very low momenta we have found a rather large value of $m_- \gtrsim 10TeV$. If the chiron mass term is only moderately momentum dependent the direct chiron effects for the LHC would be rather small - the chiron mass would remain above the momentum range reached by the LHC. In this case the dominant signature of chiral freedom for the LHC would be the non-perturbative effective Higgs-sector, with typical masses for the composite scalar fields in the 500 GeV range.

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